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COMPUTER MODEL FOR THE QUANTUM UNITARY LINEAR TRANSFORMATION OPERATOR IN MATHEMATICA SOFTWARE*

В пакете программ «Mathematica» разработана схема построения эрмитового квантового оператора, изменяющего ориентацию квантового вектора состояния. Протестирован набор доступных программных инструментов на примере анализа данных нейтронного монитора. Спектр собственных значений этого оператора сравнивается с исходными данными.
Ключевые слова: квантовый алгоритм, квантовый компьютер, унитарное преобразование, матрица поворота, нейтронный монитор.

Mathematica бағдарламалар пакетінде кванттық күй векторының бағытын өзгертетін эрмитті кванттық операторды құру схемасы әзірленді. Нейтрондық монитордың мәліметтерін талдау мысалында қолжетімді бағдарламалық құралдардың жиынтығы сынақтан өткізілді. Осы опёратордың меншікті мәндерінің спектрі бастапқы мәліметтермен салыстырылды.
Туйінді сөздер: кванттық алгоритм, кванттық компьютер, унитарлы түрлендіру, бұрылу матрицасы, нейтрондық монитор.

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Software package Mathematica was used to develop a scheme for creation of Hermitian quantum operator that alters orientation of quantum state vector. A set of available software tools was tested on the basis of neuronal monitor data analysis. Eigenvalue spectrum of this operator is compared with the original data.
Key words: quantum algorithm, a quantum computer, a unitary transformation, the neutron monitor, neutron monitor.

Since the very first moment, when the idea of the quantum computer saw the light [1], it is believed to be able to solve the new complex computational problems or provide linear, polynomial or even faster speedup for the existing ones [2]. Today, after some time has passed and numerous advancements have being made, it is somewhat hard to come up with the new quantum algorithms. Theoreticians, who are not working with the optical, solid-state or other hardware implementation of a quantum computer, are studying the Hamiltonian or the quantum states entanglement and its time evolution specific for the different quantum systems, real or hypothetical [3]. We chose to look into the applied tasks of interpreting time series data statistics in terms of parameters specific to the quantum computations, namely unitary transformation matrices and their eigenvalues.

Previously we implemented several basic quantum algorithms in Matlab [4]. This package has very versatile matrix algebra tools and libraries though it is not unique on the market and not always available for some reasons. Mathematica package is able to perform complex mathematical operations and transformation in analytical and numerical form and with high efficiency as well. This efficiency we want to harness and exploit in this work.

Methods. If we chose to skip the binary form of the data representation and handle the data sequence in our model as the quantum state vector's probability amplitudes the action of the quantum computer gate is still described by a unitary transformation.

In the previously studied case of relating two n-qubit long quantum states to each other by a unitary transformation [4] we connected these states by the numerically calculated, unitary, real transformation matrix sampled randomly from the SO(n) group. The path con-

necting these two orientations of the quantum vector in n-dimensional space is not unique, unless we have some constrains imposed on the Hamiltonian of the quantum system under consideration.

In this work, we explored the possibility of alternative approach to this procedure by direct analytical computations of a single rotation taking place in the plane connecting these two state vectors. This task is effectively addressed in Mathematica software using RotationMatrix command.

We took one minute resolution, 398654 counts long, single channel data from the 18NM64 neutron monitor hosted at Tian-Shian high elevation research station, 3340 m above the sea level [5]. Our particular data piece is sampled from 28/08/2013:00.00.00 UTC (universal time) and until 21/09/2013:23.59.00 UTC. As usual, data are corrected for the atmospheric pressure.

Two equally sized data pieces $S_1(t)$ and $S_2(t)$, each n counts long, were taken from this original signal, see the shaded areas on the inset to Figure 1c. Neutron's counts were assigned to the corresponding state vectors $\psi_1=\Sigma a_i|k_i>$ and $\psi_2=\Sigma b_j|k_j>$ as the probabilities amplitudes ai and b_j. These individual neutron counts per minute were normalized before the assignment in such a way that $\Sigma a_i^2=\Sigma b_j^2=1$ as expected from the quantum state vector probabilities amplitudes. These two pieces of data are separated by a certain distance D, from tail to tail, and may overlap (if $n>D$).

The single set of nonzero eigenvalues is obtained by calculating the rotation matrix R_θ=RotationMatrix[{ψ_1,ψ_2}], connecting these two states, and subsequent calculation of Eigenvalues[R_θ]. In order to accumulate eigenvalues statistics, we move, each time by one count, this couple of the acquisition windows through the whole data sequence until the whole data set is covered. The single window size is n=100 and the distance is D=16. Both of them were kept constant for this particular piece of data.

Results. We have plotted our results on the Figure 1. Top row, from left to the right, displays the histograms of original data, of the eigenvalues' arguments distribution, and spacing distribution between the ordered in the ascending fashion eigenvalues. Right below these pictures, we can see the cumulative distribution functions (CDF) func-

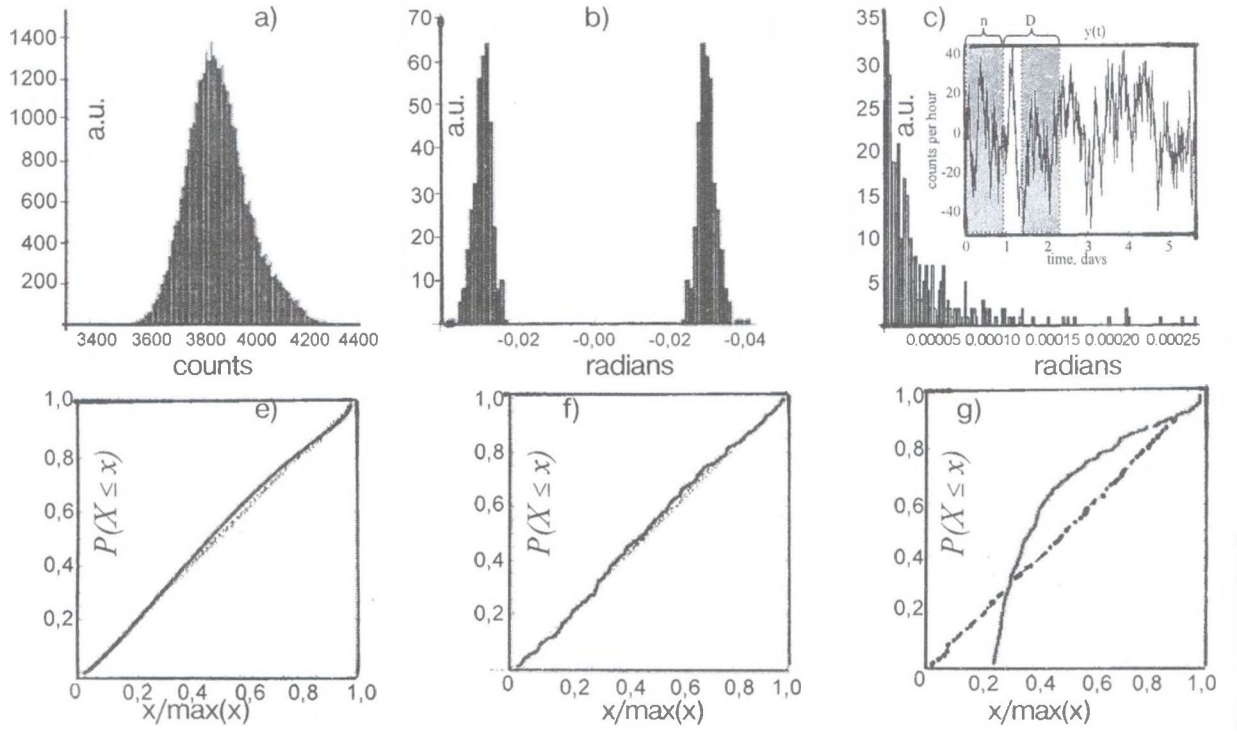

Figure 1. (a) Original data distribution. (b) Eigenvalues' arguments distribution. (c) Distribution of the spacing between the eigenvalues. Inset: The original neutron monitor data sampled by the two different state vectors (shaded by grey color). (e)-(f) Corresponding cumulative distribution functions (CDF) plotted against the normal distribution (straight line). In case of eigenvalues' distribution, only the positive values are used for CDF plot.

tions for each data sets. Taking into account that the Eigenvalues[R_θ] procedure produces the couple of eigenvalues symmetrical about zero point, we use only the positive values for the further analysis and comparison.

Discussions. The theory predicts that we should have two nonzero eigenvalues of the rotation matrix R_θ, namely $\{e^{\pm i\theta}\}$, and indeed, we observe two distinct peaks on Figure 1(b). Original data statistics, as well as eigenvalues' distribution from the right peak, as we can see it, are just slightly different from the normal distribution and bear the close resemblance to each other. The last observation does not require an extensive theoretical proof as one expect such a behavior from the algebra of statistics of two vectors ψ_1 and ψ_2 [6]. That is in the operator expression $\psi_1 \cdot R_\theta = \psi_2$, describing the result of application of the rotation operator R_θ, the eigenvalues' statistic should reflect the statistical properties of the original data. Eigenvalues' spacing, that is the distance between the sorted eigenvalues, awaits the further proper application and explanation in the analysis data statistics and has roughly exponential (probably multiexponential) type. Overall procedure is fast and takes time roughly proportional to the square of the vector's ψ length.

Concussions. Nowadays, computers and software packages are taking the tedious job of data analysis out of our hands and providing the incredible tools for physical modeling and discovery. Neat numerical experiments are conducted on a desktop PC to explore the possibilities of the quantum computations. We have studied the simple algorithm for the construction of the rotation matrix in Mathematica software. This one represents the unitary rotation operator in n-dimensional Hilbert space, connects two arbitrary quantum states constructed from our data, and could be used further for the development of different algorithms.

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