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DERIVATION AND ANALYSIS OF THE DYNAMIC EQUATIONS OF MOBILE ROBOTS WITH RANDOM DISTURBING FORCES BASED ON THE PRINCIPLE OF LEAST GAUSS FORCE

Tuleshov A.K.[1], Seidakhmet A.Zh.[1], Abduraimov A.E.1, Kamal A.N.[1]

[1]Institute of mechanics and engineering named after U.A. Dzholdasbekov, Almaty, Kazakhstan

Abstract. In this paper, the derivation of the equations of dynamics of a four-wheeled mobile robot is carried out using the variational principle of least constraint, known as the Gauss principle. Equations of nonholonomic constraints are obtained. The function of the measure of coercion of the four-wheeled mobile robot is composed. Dynamic equations based on the Gauss principle are obtained taking into account the dynamic characteristics of two DC motors. Methods for taking into account the friction forces on the wheels and random perturbations due to the unevenness of the canvas are proposed. On the Maple platform, an algorithm and a program for modeling the dynamics of a mobile robot based on the Gauss principle were developed the correctness of the obtained equations of robot motion were proved.

Key words: *Mobile wheeled robot, Gauss principle, equations of dynamics, motion modeling, disturbing forces.*

Introduction. The subject of the proposed study is a four-wheeled cargo mobile transport robot (CMTR). Such robots are used in the machine-building complex for flexible automated production tasks[1], warehouse terminals, and are gaining an increasing sector in the mining industry [2].

The research begins with modeling the dynamics of mobile robot (MR). A lot of works are devoted to solving this problem[1]. Algorithms for implementing dynamic calculations can be built using traditional Lagrange-Euler or Newton-Euler methods [3]. In many works, other forms of robot motion equations are used. These include the Wicker equations [4], or the recurrent Hollerbach equations [5], obtained using the Lagrange-Euler method; the Lu equations [6] based on the Newton-Euler method; the Lee equations [7] using the generalized Dalembert equations. All these equations are different in form, since they are obtained for different purposes [1,3]. Some of them provide the minimum time for calculating control moments and reactions in the joints of the manipulator, others are used in the synthesis and analysis of control laws, the third is used to simulate manipulator movements [8].

In the dynamics of wheeled MR, the main issue of modeling is the interaction of the wheel with the surface (relief), which is characterized as a non-holonomic bond [9] or the friction force based on the Coulomb-Amonton law [10,11,12], with liquid friction according to the Newton formula [12]. In [11], four possible cases of wheel-terrain interaction were noted. The first case is a rigid wheel moving over rough terrain. The second case is a rigid wheel moving over deformable terrain. The third case is a deformable wheel moving over a deformable terrain. The fourth case is a deformable wheel moving over rough terrain. Although many different types of models (i.e. finite elements, discrete elements, empirical) have been developed for each of these four cases, the focus here is on analytical models [12]. At the same time, in these works, the random nature of friction associated with the change of sign and the consideration of abrupt changes in the load were ignored. For highly loaded robots, dynamic performance indicators become essential because they have a significant impact on the modes of movement of the system.

Relevance in the design of a transport mobile robot is the description of its dynamics under the influence of external disturbances of a stochastic nature on the cart, the study of the robot's response to such disturbances (a jump-like change in the gravity of the load) and the consideration of nonholonomic constraints (wheel slippage).

In this regard, the practical application of a mobile transport robot requires extensive research of the dynamics and evaluation of the accuracy of the quality of movement in order to optimize the developed control system, while not changing the developed concept and hierarchical structure of the intelligent control system MR [13,14,15].

The novelty of this article is the differential equations of motion of a mobile robot obtained using the variational principle of least constraint, known as the Gauss principle.

Algorithms and numerical programs have been developed for analyzing and deriving calculated formulas of disturbing forces, including stochastic ones, due to random obstacles under the wheels, abrupt changes in the load and its movement on the upper platform, sudden changes in the directions of movement, acceleration and braking of the robot in a short period of time.

Derivation of dynamic equations. The universal platform of the mobile robot consists of a frame on which four wheels and two electric motors are attached (Figure 1). The two rear wheels are driving. The robot platform is a frame of variable length on which various mechanisms can be installed.

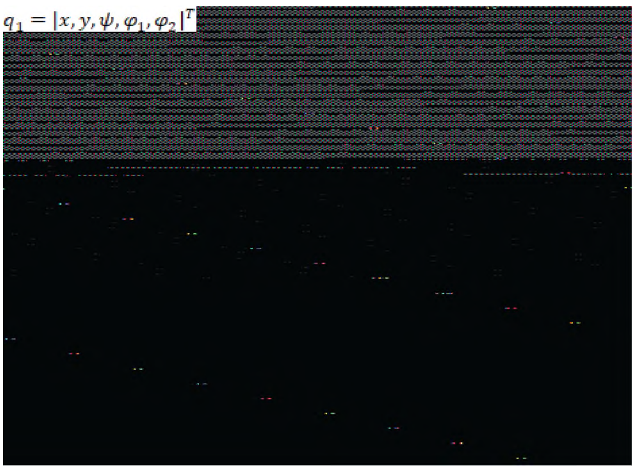


Figure 1 - Calculation scheme of a mobile robot for deriving equations of motion

We introduce the following coordinate systems: a fixed coordinate system $Oxyz$, the plane of which Oxy coincides with the horizontal rough plane on which the wheels of the robot roll, and the movable system $Ax_1y_1z_1$ starting at point A, rigidly connected to its platform (Figure 1). At the same time, the axis Ay_1 directed along the line C_3C_4, and the center of gravity of the robot C_1 lies on the axis Ax_1, being the axis of symmetry of the chassis.

When modeling the movement of a mobile robot, we introduce a number of conditions: a) the robot is considered as a system of absolutely rigid bodies; c) the movement is carried out without slipping; d) the masses of the front wheels, gears of reduction gears are considered equal to zero; c) the robot moves with the driven wheel forward.

Communication equations. The position of the bodies of the mobile robot in the coordinate system $Oxyz$ is determined by the vector of generalized coordinates $q_1 = |x, y, \psi, \varphi_1, \varphi_2|^T$, where x, y – координаты точки A – the midpoints of the segment connecting the centers C_3C_4 rear wheels 3,4; ψ - angle of rotation around the vertical platform 1, measured from the axis Ox; φ_1, φ_2 - angles of rotation of the driving wheels relative to the horizontal axes. Accordingly, the vector of generalized robot velocities has the form $\dot{q} = |\dot{x}, \dot{y}, \dot{\psi}, \dot{\varphi}_1, \dot{\varphi}_2|^T$.

The platform angular velocity vector is defined as $\Omega = |0,0,\dot{\psi}^{\cdot}|^T$, where vector Ω given by projections on the axes $Axyz$. The vectors of the angular velocities of the drive wheels are determined by the relations: $\Omega_1 = |0, \dot{\varphi}_1, \dot{\psi}^{\cdot}|^T$, $\Omega_2 = |0, \dot{\varphi}_2, \dot{\psi}^{\cdot}|^T$, where Ω_1, Ω_2 given as projections on the axes $Ax_1y_1z_1$.

The speeds of the points of contact of the driving wheels with the surface can be determined from the equations:

$$V_{P_3} = V + [\Omega, l] + [\Omega_1, r], \quad (1)$$

$$V_{P_4} = V + [\Omega, l] + [\Omega_2, r],$$

where V - linear velocity vector of point A of the platform; V_{P_3} - contact point velocity vector P_3 left wheel; V_{P_4} - contact point velocity vector P_4 right wheel; $l = AC_3 = AC_4$ – half the distance between the driving wheels; $r = C_3P_3 = C_4P_4$ – drive wheel radius.

Since the movement of the drive wheels occurs without slipping, it means $V_{P_4} = V_{P_3} = 0$. Taking into account this condition, based on the projection of equation (1) on the axis $Ax_1y_1z_1$ we obtain three independent equations of non-integrable (non-holonomic) constraints:

$$\begin{cases} V_{P3y1} = V_{P4y1} = -\dot{x}\sin\psi + \dot{y}\cos\psi = 0, \\ V_{P3x1} = \dot{x}\cos\psi + \dot{y}\sin\psi + l\dot{\psi} - r\dot{\phi}_1 = 0, \\ V_{P4x1} = \dot{x}\cos\psi + \dot{y}\sin\psi - l\dot{\psi} - r\dot{\phi}_2 = 0, \end{cases} \quad (2)$$

Vector of pseudovelocities ($\dot{\pi} = [V\ \Omega]$) includes two elements: velocity $V = \dot{x}\cos\psi + \dot{y}\sin\psi$ points A, angular velocity of platform $\Omega = \dot{\psi}$. The relationship between the generalized and pseudovelocities of the system in this case has the form

$$\dot{q} = H\dot{\pi} \quad (3)$$

Let's write down the matrix H:

$$H = \begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \\ \frac{1}{r} & \frac{l}{r} \\ \frac{1}{r} & -\frac{l}{r} \end{bmatrix}$$

Dependence (3) between generalized velocities and pseudovelocities can be rewritten in scalar form:

$$\dot{x} = V\cos\psi,\ \dot{y} = V\sin\psi, \quad (4)$$
$$\dot{\psi} = \Omega,\ \dot{\varphi}_1 = \frac{V+l\Omega}{r},\ \dot{\varphi}_2 = \frac{V-l\Omega}{r},$$

Dynamic equations. The derivation of differential equations will be carried out using the variational principle of least constraint, known as the Gauss principle. As a measure of coercion, a value Z is taken in the form of the following functional

$$Z = \frac{1}{2}\sum_{i=1}^{N}\left[m_i\left(\ddot{x}_i - \frac{F_{ix}}{m_i}\right)^2 + m_i\left(\ddot{y}_i - \frac{F_{iy}}{m_i}\right)^2 + J_i\left(\ddot{\phi}_i - \frac{M_i}{J_i}\right)^2\right]. \quad (5)$$

Here F_{ix}, F_{iy} - projections of external forces reduced to the center of mass, M_i - moment of external forces, m_i, J_i - mass and moment of inertia relative to the center of mass of the *i-th* link, $\delta\ddot{x}_i, \delta\ddot{y}_i, \delta\ddot{\phi}_i$ - variations of projections of the acceleration vector and angular acceleration.

The equations of dynamics of a mechanical system are obtained from the stationarity condition in variational form and the necessary conditions for the minimum of the functional (5)

$$\delta Z = 0, \quad \frac{\partial Z}{\partial \ddot{\pi}} = 0. \quad (6)$$

The moving parts of the mobile robot are the platform and wheels, which, relative to the plane of their location, make flat movements. The following designations are accepted: m_1 – суммарная масса платформы, J_1 - the moment of inertia of the robot about the vertical axis passing through its center of mass C_1, $a = AC_1$ – distance from point A to the center of gravity of the robot C_1, m_k – total weight of the driving wheel, J_y – moment of inertia of the wheel about the horizontal axis.

Then functional (5) for the considered mobile robot can be written in the form

$$Z = \frac{1}{2}\left[m_1\left(\dot{V} - \frac{F}{m_1}\right)^2 + (J_1 + m_1a^2)\left(\ddot{\psi} - \frac{M_R}{J_1+m_1a^2}\right)^2 + (J_y + m_kr^2)\left(\ddot{\varphi}_1 - \frac{M_{fr_1}-M_{d1}}{J_y+m_kr^2}\right)^2 + (J_y + m_kr^2)\left(\ddot{\varphi}_2 - \frac{M_{fr_2}-M_{d2}}{J_y+m_kr^2}\right)^2\right], \quad (7)$$

where M_{fr1}, M_{fr2} — moments of rolling friction on the driving wheels; M_{d1}, M_{d2} – driving moments; F – projection of the main force on the direction of velocity V, brought to point A platform, M_R – main moment of forces acting on the platform.

In equation (7) from system (4), we substitute the last two equations, which are presented in the form

$$\ddot{\varphi}_1 = b_1\dot{V} + b_2\dot{\Omega},$$
$$\ddot{\varphi}_2 = b_1\dot{V} - b_2\dot{\Omega}, \quad (8)$$

where $b_1 = {}^1/_r$, $b_2 = {}^l/_r$.

Then

$$Z = \frac{1}{2}\ m_1\left(\dot{V} - \frac{F}{m_1}\right) + (J_1 + m_1a^2)\left(\dot{\Omega} - \frac{M_R}{J_1+m_1a^2}\right) + (J_y + m_kr^2)\left(b_1\dot{V} + b_2\dot{\Omega} - \frac{M_{fr_1}-M_{d1}}{J_y+m_kr^2}\right)^2 + (J_y + m_kr^2)\left(b_1\dot{V} - b_2\dot{\Omega} - \frac{M_{fr_2}-M_{d2}}{J_y+m_kr^2}\right)^2\Big], \quad (9)$$

From conditions (6) one can obtain four equations. The first condition satisfies the equation

$$\delta Z = B_1(\ddot{\pi}_1, \ddot{\pi}_2)\delta\ddot{\pi}_1 + B_2(\ddot{\pi}_1, \ddot{\pi}_2)\delta\ddot{\pi}_2 = 0. \quad (10)$$

Note that synchronous variation takes place here, in which only the acceleration remains $\vec{v}_{i1} = \vec{v}_{i2}$, $\vec{w}_{i1} \neq \vec{w}_{i2}$, which is called Gaussian variation:

$$\delta\vec{r}_i = \frac{1}{2}\delta\vec{w}_i(\Delta t)^2, \quad (11)$$

where Δt - short time, $\delta \vec{r}_i$ - displacement vector variation, $\delta \vec{w}_i$ - acceleration vector variation.

Taking into account the independence of pseudoaccelerations $\ddot{\pi}_1 = \dot{V}$ и $\delta\ddot{\pi}_2 = \dot{\Omega}$ to fulfill equation (10), it is necessary that

$$B_1(\ddot{\pi}_1, \ddot{\pi}_2) = 0, B_2(\ddot{\pi}_1, \ddot{\pi}_2) = 0. \quad (12)$$

Equation (12) is used to determine the driving forces M_{d1}, M_{d2}.

The equations of motion of the mobile robot will be obtained from the equations

$$\frac{\partial Z}{\partial \ddot{\pi}_1} = 0, \qquad \frac{\partial Z}{\partial \ddot{\pi}_2} = 0 \quad (13)$$

Let us assume that DC motors are installed on the driving wheels [13,17]. Then, based on equations (12) and (13), we obtain the equations of the dynamics of a mobile robot in the following form:

$$\begin{cases} m\dot{V} = \frac{nc}{r}(i_1 + i_2) - \mu_n V + \frac{1}{r}(M_{fr1} + M_{fr2}) + m_1 a\Omega^2 \\ J\dot{\Omega} = \frac{ncl}{r}(i_1 - i_2) - \mu_B \Omega + \frac{l}{r}(M_{fr1} - M_{fr2}) - m_1 aV\Omega \\ L\frac{di_1}{dt} + Ri_1 + \frac{nc}{r}(V + l\Omega) = U_1 \\ L\frac{di_2}{dt} + Ri_2 + \frac{nc}{r}(V - l\Omega) = U_2 \end{cases} \quad (14)$$

where L- inductance; i_1, i_2 - currents in the armature circuits; R – armature circuit resistance; U_1, U_2 – circuit voltage (control parameters); n - gear ratio.

The coefficient of electromechanical interaction with is determined as follows:

$$c = \frac{(M_1 - M_2)U_H}{\dot{\gamma}_H M_1}, \quad (15)$$

where M_1 - motor starting torque; M_2 - rated motor torque; $\dot{\gamma}_H, U_H$ - respectively, the rated angular velocity and the rated voltage of the electric motor.

Values M_{frk} (k= 1,2) from equations (14) we define as follows:

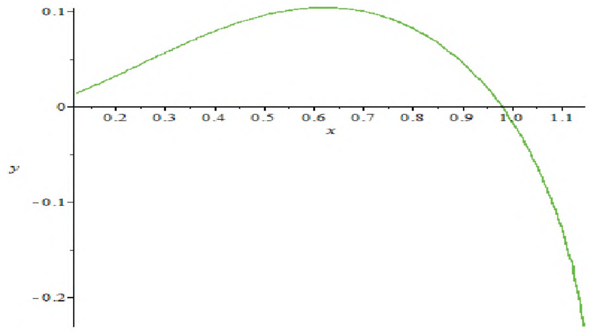
$$M_{frk} = \begin{cases} -\delta N_k sign(\dot{\varphi}_k), \dot{\varphi}_k \neq 0, \\ -nci_k, \dot{\varphi}_k = 0, |nci_k| \leq \delta N_k, \\ -\delta N_k sign(i_k), \dot{\varphi}_k = 0, |nci_k| > \delta N_k \end{cases} \quad (16)$$

where δ — коэффициент трения качения; N_k — normal reaction force of the horizontal reference plane acting on k - driving wheel.

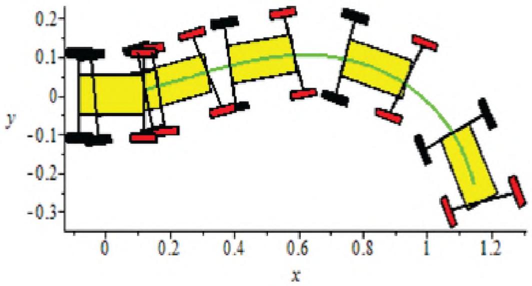
Discussion of results and conclusion. As a result, based on the Gauss principle of least constraints, the equations of dynamics of a four-wheeled mobile robot with two driving wheels are obtained. Equations (14) take into account the moments of friction force that occur between the wheels and the web, as well as the dynamic characteristics of DC motors. The unevenness of the web when modeling the dynamics of a mobile robot is carried out by adding to the system (14) the following equation

$$m\ddot{z} = -c(z - h) - \alpha(\dot{z} - \dot{h}), \quad (17)$$

where the functions h(z) of the road roughness and has the form of a function with a random amplitude.



a)



b)

Figure 2 – Graphs of the movement of a mobile robot:
a) the trajectory of movement;
b) animation of moving the robot platform

In the Maple analytical computing system, a program was compiled for simulating the movement of a mobile robot based on equations (14) and calculating the transverse vibrations of the robot body when moving along a road with bumps based on equation (17). Figure 2 shows the simulation results. Figure 2a shows the trajectory of the center of gravity of the mobile robot with the speed V and the angular velocity of rotation Ω of the platform relative to this center. The turn can be clearly seen in Figure 2b, which shows the animation of platform movements along the center trajectory. Figure 3 shows a plot of the speed V of the platform along the trajectory. The movement speed is controlled by changing the control parameters U_1, U_2.

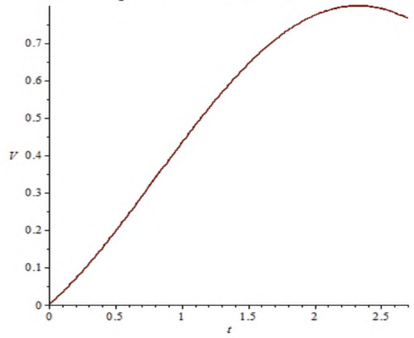


Figure 3 - Graph of the change in the speed V of the platform along the trajectory

The segments of the piecewise linear characteristic determine the number of segments that are involved in the fitting procedure. The fitting method is an exact method for solving a particular equation.

In the Maple system, a program was compiled for solving the differential equation of a mobile robot, taking into account (17). Let's take the coefficient of road resistance ?

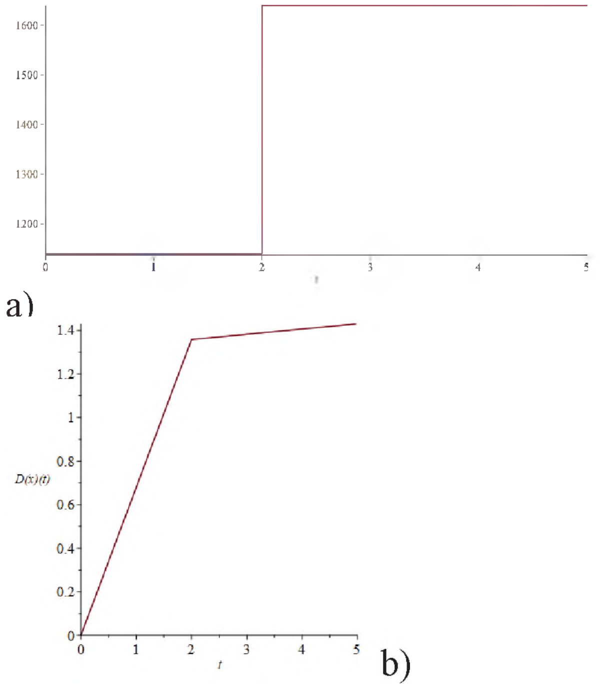


Figure 4 - Simulation of the dynamics of the robot, taking into account the unevenness of the web: a) a graph of the change in the function h(z); b) the graph of the speed during the acceleration of the mobile robot for 5sec and the abrupt change in the load at t = 2 c

Graph 4b shows the acceleration rate of the mobile robot for 5 seconds. As can be seen from the graph, after the mass was added (t = 2 s), the acceleration speed became slow.

In addition, the value of the functional is indirectly related to the reactions in kinematic pairs according to the equality obtained from the Gauss principle

$$Z = \frac{1}{2}\sum_{i=1}^{N}\frac{R_i^2}{m_i} \quad (18)$$

The condition that the quantity is minimal for actual motion leads to an extreme property of constraint reactions: for actual motion, constraint reactions are minimal.

For example, in order to estimate the main vector of force and the main moment of forces reduced to the center of gravity of the platform, we have:

$$Z = \frac{1}{2}\left[\frac{R_{12}}{m_1} + \frac{M_{12}}{J_1 + m_1 a^2}\right], \quad (19)$$

where R_{12} – the reaction of the connection between the platform and the wheels, reduced to a point A, M_{12}- moment of coupling reactions between the platform and wheels, relative to the point A.

To find R_{12} и M_{12} needs to (7) be shaped for (19).

Thus, the Gaussian principle of no less compulsion made it possible to simplify the procedure for deriving the equations of the dynamics of a mobile robot, taking into account the perturbing forces, and to evaluate the reactions between individual moving parts (links) of the robot. The equations of dynamics obtained on the basis of the Gauss principle are correct and make it possible to simulate the motion of the MR taking into account random perturbing forces.

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А.К. Тулешов, А.Ж. Сейдахмет, А.Е. Абдураимов, А.Н. Камал. ВЫВОД И АНАЛИЗ УРАВНЕНИЙ ДИНАМИКИ МОБИЛЬНЫХ РОБОТОВ С УЧЕТОМ СЛУЧАЙНЫХ ВОЗМУЩАЮЩИХ СИЛ НА ОСНОВЕ ПРИНЦИПА НАИМЕНЬШЕГО ПРИНУЖДЕНИЯ ГАУССА

Аннотация. В работе вывод уравнений динамики четырехколесного мобильного робота осуществляется с использованием вариационного принципа наименьшего принуждения, известного как принцип Гаусса. Получены уравнения неголономных связей. Составлена функция меры принуждения четырехколесного мобильного робота. Уравнения динамики на основе принципа Гаусса получены с учетом динамической характеристики двух двигателей постоянным током. Предложена методики учета сил трения на колесах и случайных возмущений за счет неровности полотна. На платформе Maple разработан алгоритм и программа моделирования динамики мобильного робота на основе принципа Гаусса и доказана корректность и правильность полученных уравнений движения робота.

Ключевые слова: Мобильный колесный робот, принцип Гаусса, уравнения динамики, моделирование движения, возмущающие силы.

А.К. Тулешов, А.Ж. Сейдахмет, А. Е. Абдураимов, А.Н. Камал. ЕҢ АЗ ШЕКТЕУ ГАУСС ПРИНЦИПІ НЕГІЗІНДЕ КЕЗДЕЙСОҚ ҚОЗДЫРҒЫШ КҮШТЕРДІ ЕСКЕРЕ ОТЫРЫП, ЖЫЛЖЫМАЛЫ РОБОТТАРДЫҢ ДИНАМИКАСЫНЫҢ ТЕҢДЕУЛЕРІН ШЫҒАРУ ЖӘНЕ ТАЛДАУ

Түіндеме. Бұл жұмыста төрт доңғалақты жылжымалы роботтың динамикасының теңдеулерін шығару Гаусс принципі деп аталатын ең аз шектеудің вариациялық принципін қолдану арқылы жүзеге асырылады. Голономдық емес шектеулердің теңдеулері алынды. Төрт доңғалақты мобильді роботтың мәжбүрлеу функциясын құрастырылды. Гаусс принципіне негізделген динамикалық теңдеулер тұрақты токтың екі қозғалтқышының динамикалық сипаттамаларын ескере отырып алынады. Дөңгелектердегі үйкеліс күштерін және кенептің кедір-бұдырлығына байланысты кездейсоқ бұзылуларды есепке алу әдістері ұсынылған. Maple платформасында Гаусс принципі бойынша жылжымалы роботтың динамикасын модельдеу алгоритмі мен бағдарламасы жасалып, роботтың қозғалыс теңдеулерінің алынған дұрыстық мен дұрыстығы дәлелденді.

Түйінді сөздер: Қозғалмалы доңғалақты робот, Гаусс принципі, динамика теңдеулері, қозғалысты модельдеу, кедергі күштері.

Сведения об авторах

Тулешов Амандык Куатович, доктор техн. наук, профессор, член-корреспондент НАН РК.
e-mail: aman_58@mail.ru:
Сейдахмет Аскар Жунусович, канд.техн.наук, доцент.
e-mail: seydakhmet@mail.ru:
Абдураимов Азизбек Гралиевич, магистр естественных наук.
e-mail: zizo_waterpolo@mail.ru:
Камал Азиз Нутпулла-оглы, магистр естественных наук.
e-mail: kan77705@gmail.com: